CHAPTER IV

Group Theory, Matrix Algebra and Coding Theory

Solutions for the Exercises of Section IV.5

This is a corrected solution for Question IV.5.23.

23. Let $H = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ be the parity-check matrix for a group code C.

a) Give the generator matrix G for C.

Solution. $G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$.

b) List the elements of C.

Solution.

10001110	01001101	00101011	00010111
11000011	10100101	10011001	01100110
01011010	00111100	11101000	10110010
11010100	01110001	11111111	00000000

c) What is $\mu(C)$? Determine that C is single-error correcting, but not double-error correcting.

Solution. Every nonzero code word has weight 4, except for 11111111, which has weight 8. Thus $\mu(C) = 4$. Thus $2k + 1 < 4$ if and only if $k < 1.5$; that is, if and only if $k \leq 1$. Thus C will allow us to correct every occurrence of a single error, but not every occurrence of two errors.

d) For each of the given syndromes, provide a coset leader.

Syndrome	Coset leader
0000	**00000000**
1000	**00001000**
0100	**00000100**
0010	**00000010**
0001	**00000001**
1110	**10000000**
1101	**01000000**
1011	**00100000**
0111	**00010000**

e) Of the 7 syndromes that do not appear in the table above, we have chosen two and presented them in the table below. For each syndrome listed below, give two error words of weight 2, each of which would produce the given syndrome.

Syndrome	First word	Second word
1111	**00011000**	**00100100**
1010	**10000100**	**00001010**

f) Suppose that a code word α is transmitted and received as $\gamma = \alpha + \varepsilon$ where $\gamma = \mathbf{10101101}$.

(i) Use the syndrome-coset leader table that you completed above to correct the error and determine α.

Solution. The syndrome of **10101101** is $\mathbf{1110} + \mathbf{1011} + \mathbf{1000} + \mathbf{0100} + \mathbf{0001} = \mathbf{1000}$. The error word for the syndrome **1000** is **00001000**, so the transmitted code word was $\mathbf{10101101} + \mathbf{00001000} = \mathbf{10100101}$.

(ii) Extract the data word that was encoded in α.

Solution. The data word is embedded in the code word as the first four bits, so it is **1010**.