8 marks 1. Find the Taylor series for $f(x) = \sin x$ centered at $a = \frac{\pi}{2}$. You do not need to determine its radius or interval of convergence.

6 marks 2. (a) Find the Maclaurin series for $\ln(1 + x^2)$.

5 marks (b) Find the first three non-zero terms in the Maclaurin series for $\ln^2\left(1+x^2\right)$. Note that $\ln^2(z) = [\ln(z)]^2 = [\ln(z)]\,[\ln(z)]$.

10 marks 3. Estimate $\displaystyle\int_0^1 \frac{e^{-t}-1}{t}\,dt$ with an error that does not exceed 0.01.

Please leave your answer as a sum of fractions.

6 marks 4. Compute the sum of the series

$$\frac{\pi^2}{2!} - \frac{\pi^4}{4!} + \frac{\pi^6}{6!} - \frac{\pi^8}{8!} + \dots \quad .$$

Hint: Plug an appropriate value of x into an appropriate Maclaurin series.

5. Consider the following parametric equations

(i) $x(t) = t, \qquad y(t) = t^2 - 2t + 1, \qquad 0 \le t \le 2.$

(ii) $x(t) = 2 - t, \qquad y(t) = t^2 - 2t + 1, \qquad 0 \le t \le 2.$

(iii) $x(t) = 2t, \qquad y(t) = 4t^2 - 4t + 1, \qquad 0 \le t \le 1.$

4 marks (a) Each set of equations traces out the same Cartesian curve. Sketch this common curve.

4 marks (b) What is the difference between the manner in which curves (i) and (ii) are traced out?

4 marks (c) Find the equation of the line which is tangent to curve (iii) when $t = \frac{3}{4}$.

6. Let C be the curve given parametrically by

$$x = \cos t + t \sin t, \qquad y = \sin t - t \cos t, \qquad 0 \le t \le \pi .$$

6 marks (a) Compute the length of C.

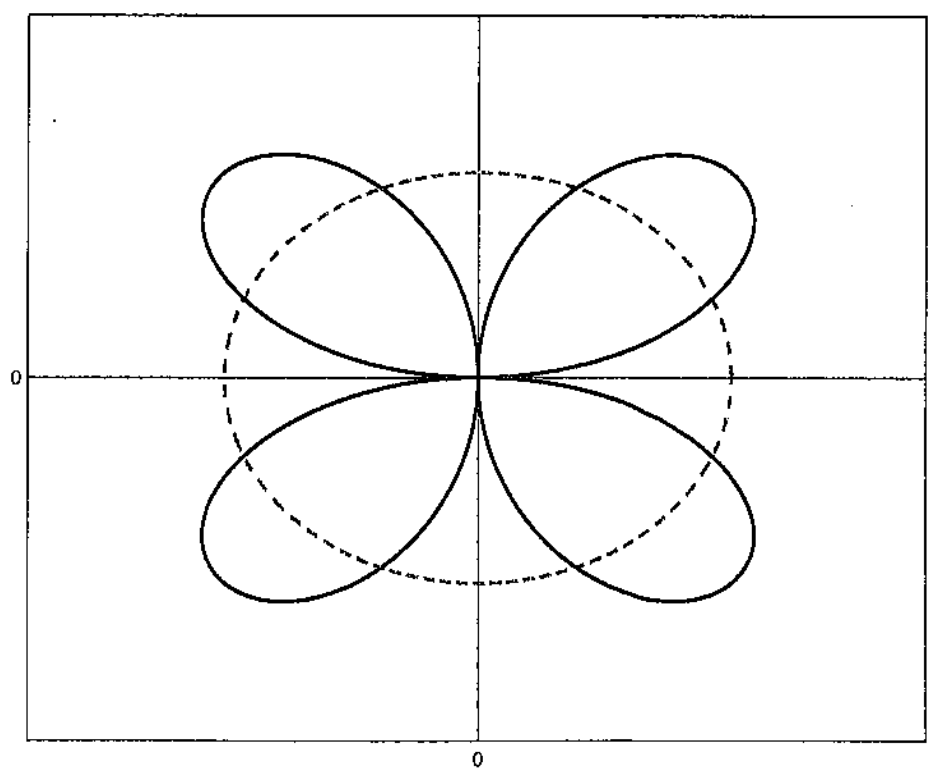
6 marks (b) Find the unique (Cartesian) point on C at which the tangent is vertical.

6 marks 7. (a) Find a general solution to the equation $y' = ty^2$.

4 marks (b) Use your answer from (a) to solve the initial value problem $y' = ty^2$, $y(0) = \frac{1}{2}$.

8 marks 8. Find the length of and area enclosed by the polar curve $r = \sin\theta$. You may use the fact that $\cos^2\theta = \frac{1}{2}\left[1 + \cos(2\theta)\right]$.

9 marks 9. The graphs of the polar curves $r = \frac{1}{\sqrt{2}}$ and $r = \sin 2\theta$ are pictured below. Let B denote the region in the first quadrant which lies between both curves. Express the area of B as the sum of three integrals. Do not evaluate any of the three integrals.



8 marks 10. Determine whether the improper integral $\int_0^1 \frac{1}{1-x^2}\,dx$ converges or diverges. If it converges, evaluate the integral.

6 marks 11. Suppose that a_n is bounded and b_n converges to zero. Using the formal definition of convergence, prove that $\lim_{n\to\infty} a_n b_n = 0$.

Hint: If a_n is bounded by M, how does the magnitude of $a_n b_n$ compare to that of $M b_n$?