Instructor's Name (**Print**) ______________________

Student's Name (**Print**) ______________________

Student's Signature ______________________

THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Calculus 1501B First Midterm Examination

Friday, February 10, 2012 7:00 p.m. – 9:30 p.m.

INSTRUCTIONS

1. Do not unstaple the booklet. Do not tear any pages from the booklet.
2. Questions start on Page 1 and continue to Page 10. Questions are printed on both sides of the paper. BE SURE YOU HAVE A COMPLETE BOOKLET.
3. CALCULATORS AND NOTES ARE NOT PERMITTED.
4. SHOW ALL YOUR WORK. Answer all questions in the spaces provided.
5. TOTAL MARKS = 100.

Student Number (**Print**)

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FOR GRADING ONLY

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4 marks 1. (a) State Rolle's Theorem.

4 marks (b) State the Mean Value Theorem.

6 marks (c) Use Rolle's Theorem to prove the Mean Value Theorem.
Hint: Consider the function $h(x) = f(x) - g(x)$, where $g(x)$ is the secant line connecting $(a, f(a))$ to $(b, f(b))$.

8 marks 2. Suppose f is continuous on $[0,2]$, differentiable on $(0,2)$ and satisfies $f(0)=0$, $f(2)=2$. Prove that there exists a point $x \in (0,2)$ such that $f'(x) = \dfrac{1}{f(x)}$.

Hint: Consider the function $g(x) = [f(x)]^2$.

6 marks 3. (a) Evaluate $\int \ln x \, dx$.

6 marks (b) Evaluate $\int \frac{8x-3}{x^2-x} \, dx$.

8 marks 4. Evaluate $\int e^{2x} \cos x \, dx$.

9 marks 5. Find the partial fraction decomposition of $\dfrac{8x^3+19x^2+10x+5}{(x^2+2x+1)(x^2+1)}$. Form alone is not sufficient (that is, make sure you determine the numerical values of all coefficents).

6. Assess the convergence of the following integrals. If an integral converges, either evaluate it or provide an upper bound on its value.

5 marks (a) $\displaystyle\int_1^\infty xe^{-x^2}\,dx$

5 marks (b) $\displaystyle\int_1^\infty \frac{1}{x}\,e^{-x^2}\,dx$

6 marks 7. Evaluate $\displaystyle\int_0^3 \frac{2x}{x^2-1}\,dx$.

8 marks 8. Show that $\Gamma(n+1) = n\Gamma(n)$ for any integer $n \geq 1$. Be precise (i.e. carefully justify each step/calculation). You may assume that the integral defining $\Gamma(n)$ is convergent for any integer n.

5 marks 9. (a) Use the formal definition to prove that the sequence $a_n = 3 + (-1)^n \frac{1}{n+7}$ converges to the limit $L = 3$.

5 marks (b) Use the formal definition (of an infinite limit) to prove that the sequence $a_n = \dfrac{n+1}{\sqrt{n}}$ diverges to ∞.

10. Determine whether or not the following sequences converge (you do not need to use the formal definition). If a sequence converges, evaluate its limit (state any theorems you use along the way). If a sequence diverges, explain why.

5 marks (a) $a_n = n^{1/n}$.

5 marks (b) $a_n = \dfrac{1}{\sin\left(\frac{(-1)^n}{n}\right)}$.

5 marks (c) $a_n = \dfrac{1+\cos(n)}{\ln(n)}$.

This page is left blank intentionally. It may be used for any answer which you could not fit in the space provided.