5 marks 1. Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$, and f'' is continuous. Find the value of

$$\int_1^4 x f''(x)\, dx$$

10 marks 2. Evaluate the integrals:

(a) $$\int_6^{13} \frac{dx}{2\sqrt{x+3}+x}$$

(b) $$\int_0^1 \frac{\ln x}{\sqrt{x}}\, dx$$

7 *marks* 3.

(a) $\displaystyle\sum_{n=1}^{\infty}\left(e^{1/n} - e^{1/(n+1)}\right) =$

(b) $\dfrac{1}{1\cdot 3} - \dfrac{1}{3\cdot 3^3} + \dfrac{1}{5\cdot 3^5} - \dfrac{1}{7\cdot 3^7} + \dfrac{1}{9\cdot 3^9} - ... =$

10 marks 4. Find the Maclaurin series for $f(x) = e^{2x} + \cos(3x)$ and for $g(x) = \ln(1 + 2x^2)$. Determine the radius of convergence for each.

5 marks 5. Find the Taylor series centered at $a = 5$ for $f(x) = \ln(2x + 3)$.

8 marks 6. (a) If $f(x) = \sum_{i=0}^{\infty} c_i(x-a)^i$ for $|x-a| < R$, show that

$$c_i = \frac{f^{(i)}(a)}{i!}$$

for $i = 0, 1, 2, ...$
(where $f^{(0)}(a) = f(a)$, and for $i \geq 1$ $f^{(i)}(a)$ denotes the i-th derivative of f at a).

(b) If $f(x) = 5x^{514}e^{x^2}$, find $f^{(2014)}(0)$.

6 marks 7. Solve the initial value problem:

$$\frac{dy}{dx} = \frac{xy\ \sin x}{y+1}$$

$$y(0) = 1$$

12 marks 8. *(Note: to receive any credit in this problem you must state very clearly which arc length formula you are attempting to use.)*

Find the length of each of the following curves:

(a) $y = \dfrac{x^3}{3} + \dfrac{1}{4x},\ 1 \le x \le 2$

(b) $x = 3\cos t - \cos(3t),\ y = 3\sin t - \sin(3t),\ 0 \le t \le \pi$

(c) $r = 5^{\Theta},\ 0 \leq \Theta \leq 2\pi$

7 marks 9. Sketch the curve and find the area that it encloses

$$r = 4 + 3\sin\Theta$$