

13  
marks

1.

(4 marks) (a) State the Mean Value Theorem precisely.

(4 marks) (b) Does there exist a function  $f$  which is differentiable everywhere and  $f(-1) = 4$ ,  $f(4) = 10$ ,  $f'(x) \leq 1$  for all  $x$  between  $-1$  and  $4$ ? Explain why.

(5 marks) (c) Show that any polynomial  $f(x) = ax^3 + bx^2 + cx + d$  has at most 3 roots. In other words there are at most 3 real numbers  $c_1, c_2, c_3$  such that  $f(c_1) = f(c_2) = f(c_3) = 0$ .

15 marks 2.

Evaluate:

(5 marks) (a)  $\int x^2 e^x dx$

(5 marks) (b)  $\int \ln x dx$

(5 marks) (c)  $\int \frac{1}{(1+x^2)^2} dx$

*10*  
*marks* 3.

Evaluate:

*(5 marks)* (a)  $\int_1^3 x^3 \ln x \, dx$

*(5 marks)* (b)  $\int_1^{\sqrt{3}} \arctan(1/x) \, dx$

8  
marks

4.

Write out the form of the partial fraction decomposition for the following rational functions without computing the coefficients:

(4 marks) (a)  $\frac{1 + 6x}{(x - 1)^3(2x + 5)}$

(4 marks) (b)  $\frac{3x^2 + 5x + 2}{x(x - 1)(x^2 + 2x + 10)^2}$

*12*  
*marks* 5.

Evaluate:

*(6 marks)* (a)  $\int \frac{5x + 1}{x^2 - 3x + 2} dx$

*(6 marks)* (b)  $\int \frac{x^3 + 4}{x^2 + 4} dx$

*12*  
*marks* 6.

Determine whether the following improper integrals are convergent and compute their values for the ones that are convergent:

*(6 marks)* (a)  $\int_4^{\infty} \frac{1}{(x-2)^{3/2}} dx$

*(6 marks)* (b)  $\int_0^1 \frac{\ln x}{x} dx$

10 marks 7.

Use the Comparison Theorem to determine whether the following integrals are convergent or divergent:

(5 marks) (a)  $\int_1^{\infty} \frac{3 + e^{-x^2}}{x} dx$

(5 marks) (b)  $\int_1^{\infty} \frac{x}{x^3 + 1} dx$