5 marks 1.

(a) $\Gamma(3) + \Gamma(4) =$

(b) If $x_0 > 1$ is such that $\Gamma(x_0) = \frac{3}{2}$, then

$$\frac{\Gamma(x_0 + 1)}{2x_0} =$$

4 marks 2. Prove that if $\lim_{n\to\infty} a_n = 0$ and $\{b_n\}$ is bounded, then

$$\lim_{n\to\infty} (a_n b_n) = 0.$$

4 marks 3. Use the Integral Test to determine whether the series is convergent or divergent:

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$$

7 marks 4. Determine whether the series converges or diverges. If it converges, - find its sum.

(a) $\sum_{n=3}^{\infty} (-\frac{1}{8})^n$

(b) $\sum_{n=1}^{\infty} \frac{n \arctan(\sqrt{n}+3)}{n+1}$

8 marks 5. Determine whether the series is convergent or divergent. State clearly which test you are using.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(6n^5+23)^{1/4}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-4n+5}{10n^3+n^2+4n}$

8 marks 6. Use the Alternating Series Test (if possible) to test the following series for convergence:

(a) $$\sum_{n=1}^{\infty} (-1)^n \frac{2n^2 + 5n}{4n^2 + 6n + 1}$$

(b) $$\sum_{n=1}^{\infty} (-1)^n n e^{-n}$$

12 marks 7. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{-1}{\sqrt{n+5}}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \Big(\frac{n}{n+1}\Big)^{n^2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{55}\ 25^n}{n!}$

12 marks 8. Find the radius and the interval of convergence for the following series;

(a) $\sum_{n=1}^{\infty} \frac{4^n}{\sqrt{n}}(x-5)^n$

(b) $\sum_{n=1}^{\infty} \frac{(3x-1)^n}{6^n\sqrt{n}}$

(c) $\sum_{n=1}^{\infty} \frac{n^2(x+1)^n}{2 \cdot 4 \cdot 6 \cdot \ldots \cdot (2n)}$