THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Calculus 1501B Midterm Examination 1

Friday, February 6, 2015 7:00 – 9:00 p.m.

1. (a) State the $\epsilon - \delta$ definition for the limit of a function at a point.
   (b) Use this definition to prove that
$$\lim_{x\to 3} \frac{4x-2}{5} = 2\,.$$

2. (a) State Rolle's Theorem.
   (b) Use Rolle's Theorem to prove that the polynomial $f(x) = x^2$ has precisely one real root. [*Hint: Assume for contradiction that $f(a) = 0$ for some $a \neq 0$.*]
   (c) Consider the function $f(x) = |2x - 1|$. Note that $f(-2) = f(3)$, but there is no $x \in (-2, 3)$ satisfying $f'(x) = 0$.
   Does this contradict Rolle's Theorem? (Provide an explanation for your YES/NO answer.)

3. Evaluate $\displaystyle\int \sqrt{e^x} \sin x \,\mathrm{d}x\,.$

4. Evaluate $\displaystyle\int \cos t \cos^3(\sin t)\,\mathrm{d}t\,.$

5. Evaluate $\displaystyle\int \frac{1}{x^2\sqrt{4-x^2}}\,\mathrm{d}x\,.$

6. (a) Write out the form of the partial fraction decomposition for the function
$$\frac{x^5 - 2x^4 - 7x + 2}{(x^2+3x+3)(x^2-2x-3)(x^2-1)}\,.$$
   Do NOT evaluate the coefficients.
   (b) Evaluate $\displaystyle\int \frac{3x^2+x}{x^2+x-2}\,\mathrm{d}x\,.$

7. Determine whether $\displaystyle\int_0^\infty \frac{1}{(x+1)^{3/2}}\,\mathrm{d}x$ is convergent. If so, compute its precise value.

8. Determine whether $\displaystyle\int_{-\infty}^{-1} e^{-2x}\,\mathrm{d}x$ is convergent. If so, compute its precise value.

9. Use the Comparison Theorem to determine the convergence of $\displaystyle\int_2^\infty \frac{\ln(x-1)}{x^3+\sqrt{x}}\,\mathrm{d}x.$
   Do NOT compute the precise value of the integral.

10. Is the following statement TRUE or FALSE:

    "Suppose f and g are continuous functions with $f(x) \geq g(x) \geq 0$ for all $x \geq a$. If $\int_a^\infty f(x)\,\mathrm{d}x$ is divergent then $\int_a^\infty g(x)\,\mathrm{d}x$ must be divergent as well."

    If TRUE, provide an argument or explanation. If FALSE, provide a counterexample.