THE UNIVERSITY OF WESTERN ONTARIO
LONDON CANADA
DEPARTMENT OF MATHEMATICS

Calculus 1501B Midterm Examination 2

Friday, March 13, 2015 7:00 – 9:00 p.m.

1. What is the value of the limit $\lim_{n\to\infty} \frac{2n\sin(1-n)}{3n^2+1}$?

   A: $-\frac{2}{3}$.
   B: 0.
   C: $\frac{2}{3}$.
   D: The sequence diverges.

2. What is the value of the limit $\lim_{n\to\infty} \frac{n!2^n}{n^n}$?

   A: $2e^{-1}$.
   B: $\frac{e}{2}$.
   C: 0.
   D: The sequence diverges.

3. What is the value of the limit $\lim_{n\to\infty} \frac{(n-\ln(\ln n))(n+\ln(\ln n))}{3n^2+1}$?

   A: $-\frac{2}{3}$.
   B: $-\frac{1}{3}$.
   C: 0.
   D: $\frac{1}{3}$.
   E: $\frac{2}{3}$.
   F: The sequence diverges.

4. How many terms of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{4^n n^4}$ are needed to estimate its sum with the accuracy of 3 decimal places (i.e., with error less than 10^{-3})?

   A: 1.
   B: 2.
   C: 4.
   D: 8.
   E: The series diverges.

5. What is the value of the limit $\lim_{n\to\infty} \frac{\Gamma(n+1)\cdot\Gamma(n+2)}{(\Gamma(n))^2(2n-1)^3}$?

A: -1.

B: $-\frac{1}{2}$.

C: 0.

D: $\frac{1}{8}$.

E: $\frac{1}{2}$.

F: The sequence diverges.

6. Let $a_n = \dfrac{(-1)^n}{n - \sqrt{n}}$ for $n \geq 2$. Select ALL the true statements below. (**NB:** You will lose marks for selecting contradictory statements.)

A: $\lim_{n \to \infty} a_n = 0$.

B: The limit $\lim_{n \to \infty} a_n$ does not exist.

C: The series $\sum_{n=2}^{\infty} a_n$ diverges.

D: The series $\sum_{n=2}^{\infty} a_n$ converges conditionally.

E: The series $\sum_{n=2}^{\infty} a_n$ converges absolutely.

F: The Alternating Series Theorem applies to this series.

G: The Alternating Series Theorem does not apply to this series.

H: Limit Comparison Test with the harmonic series can be used to show that the series $\sum_{n=2}^{\infty} a_n$ does not converge absolutely.

I: Comparison with a convergent p-series can be used to show that the series $\sum_{n=2}^{\infty} a_n$ converges absolutely.

7. Let R denote the radius of convergence of the power series $\sum_{n=1}^{\infty} \dfrac{(-1)^n \cdot n}{n^2 + 3} (x-5)^n$.

Select ALL the true statements below. (**NB:** You will lose marks for selecting contradictory statements.)

A: $R = 0$.

B: $R = \infty$.

C: $R = 1$.

D: Statements A, B, and C are false.

E: The series diverges at both $x = 5 - R$ and $x = 5 + R$.

F: The series converges at precisely one of the endpoints of the interval of convergence.

G: The series converges absolutely at both $x = 5 - R$ and $x = 5 + R$.

H: Alternating Series Theorem can be used to prove convergence at one of the endpoints of the interval of convergence.

I: Comparison with a convergent p-series can be used to determine convergence at one of the endpoints of the interval of convergence.

8. (a) State the formal $\epsilon - N$ definition of the limit of a sequence $\{a_n\}_{n=1}^{\infty}$.

   (b) Use this definition to prove that $\lim_{n\to\infty} \dfrac{2n+1}{5n} = \dfrac{2}{5}$.

9. (a) State the Monotone Convergence Theorem for sequences.

   (b) Let $\{s_n\}_{n=1}^{\infty}$ be the sequence of partial sums of a series $\sum_{n=1}^{\infty} a_n$ (i.e., $s_n = a_1 + \cdots + a_n$ for all $n \geq 1$). Suppose that

$$a_n > 0 \quad \text{and} \quad s_n \leq 2015 + \cos(\pi n^{2015})$$

   for all $n \geq 1$. Prove that the series $\sum_{n=1}^{\infty} a_n$ is convergent. Is it absolutely convergent?

10. Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \dfrac{2^{n^2}}{n!} (x+1)^n$. Justify your answer.

11. (a) State the Integral Test theorem for convergence of a series.

    (b) Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} ne^{-n^2}$ is convergent or divergent. Justify your answer.

12. Let $c > 0$ be such that $\Gamma(c) = \frac{\pi}{2}$. Find the value of $\dfrac{\Gamma(c+1)\cdot\Gamma(c+2)}{c^2(c+1)}$.